Soliton Excitation in ferromagnetic Nanotubes with Higher Order Single Ion Anisotropy

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Abstract

Dynamics of soliton in classical discrete Heisenberg ferromagnetic nanotubes (FN) with the effect of higher order single ion anisotropy was studied. The Hirota Bilinearization (HB) method was applied to the discrete nonlinear Schrodinger (DNLS) equation and obtains the soliton solution for FN system. The effects of exchange interaction and single ion anisotropy on soliton evolution along FN system were discussed through graphically.

1. Introduction

In recent years, magnetic nanostructures are mainly used for the memory storage application. A considerable attention has been devoted to the study of magnetic nanostructures such as arrays of rings, dots, stripes and wires. They have been extensively investigated because of their potential applications. The various geometries of this magnetic material have been studied extensively both experimentally and theoretically due to their interesting fundamental physical and potential technological application. The size and shape of the nanostructures are the important parameters which are mainly focused for the device application. The study of switching of the magnetization is important to know about the spin wave (SW) excitation behavior, not because of their role in determining the magnetic properties of their systems at low temperature, but also because they define the time scale for the dynamics of the magnetization. The investigation of SW is a powerful tool to understand the dynamical properties of nanomagnetic media. In addition, magnetic interactions such as magnetic anisotropy contributions, the homogeneity of the internal field and coupling between spin of atoms can be studied from SW. Another consequence of the non-linear nature is that, chaotic behavior is shown by large amplitude oscillations and SW behavior by small amplitude oscillations. Experimentally the SW can be conveniently understood by two basic techniques, ferromagnetic resonance (FMR) and Brillonin light scattering (BLS). Such a studies of the SW in ferromagnetic nanodots and nanowires, have revealed many interesting properties, such as the localization of modes near surfaces and quantization effects. Static and dynamic properties of such systems have been studied intensively in recent years [1–6]. Earlier, Wang et al [6] used BLS measurement to study the SW excitations in highly ordered arrays of ferromagnetic nickel nanowires. Researchers have been studied dipole-exchange SW in ferromagnetic nanowires and nanodots [7-10]. Recently, Ngugen et al used a microscopic theory for studying dipole-exchange SW in FN and they found spatial distributions as well as the frequencies of the SW mode [8]. In this paper, we have focused the effects of exchange interaction and single ion anisotropy on evolution of soliton in FN system. In Section 2, we considered the Heisenberg spin chain with higher order single ion anisotropy and derive the discrete nonlinear Schrodinger equation with the help of quasi discrete approximation. Use the Hirotabilinearization method; we constructed the one soliton solution in Section 3. The roles of spin exchange interaction on soliton excitation were discussed in Section 4 and the results were concluded in Section 5.

2. Model and Equation of Motion

The Heisenberg model based on a spin Hamiltonian is considered and it allows us to study the SW in the form of soliton in nanotubes, while the external magnetic field is at an arbitrary angle to the longitudinal axis. The modeling of nanotube is considered by taking hexagonal cross section (in the *XZ* plane) with chosen inner radii and outer radii *l* and *q* (*a* is the unit of lattice constant) as shown in Fig. (1). The spin of each cross-section layer is $N = 3[r_0(r_{0+1}) - r_i(r_{i+1})]$, which are vertically stacked (with spacing a) to form a nanotube extending in the *Y* - direction from *L/2* to *– L/2*, where *L* is the length of the macroscopically-large nanotube. Thus the Hamiltonian can be expressed generally as

$$H = -\frac{1}{2}\frac{Ja^2}{g\mu_B}\sum_i \vec{s_i}.\overrightarrow{s_{i+1}} - \mathrm{D}_1\sum_i (s_i^z)^2 - \mathrm{D}_2\sum_i (s_i^z)^4 - \frac{\eta g\mu_B H_0}{a^3}\sum_i s_i^z \quad \textbf{(1)}$$

Where s_i and s_{i+1} represents the spin at site *i* and neighbouring site *i+1* respectively. The parameter J is the spin exchange interaction of site *i* and *i+1*. D_1, D_2 are single ion anisotropy and higher order single ion anisotropy interaction. H_0 is the intensity of external applied magnetic field in the z direction and η is a geometrical factor depending on the lattice structure.

By treating the spin as the classical vectors, we can define the spin variables $S^-{}_n = (S_n^+)^*, S_n^z = \sqrt{1 - S_n^+ S_n^-}$, using classical spin vectors, we obtain DNLS equation,

$$i\hbar\frac{ds_i^+}{dt} = -\frac{1}{2}\frac{\hbar Ja^2}{g\mu_B}\Big\{(s_{i+1}^+ + s_{i-1}^+) - \frac{1}{2}|s_i^+|^2(s_{i+1}^+ + s_{i-1}^+) - s_i^+\Big[\Big(1 - \frac{1}{2}|s_{i+1}^+|^2\Big)\Big] + \Big(1 - \frac{1}{2}|s_{i-1}^+|^2\Big)\Big\}$$
$$+2D_1\Big[s_i^+ - \frac{1}{2}|s_i^+|^2\Big] + 4D_2\Big[s_i^+ - \frac{3}{2}s_i^+|s_i^+|^2 + \frac{3}{4}s_i^+|s_i^+|^4 - \frac{1}{8}s_i^+|s_i^+|^6\Big] + \frac{\eta g\mu_B H_0}{a^3}[s_i^+] \quad (\mathbf{2})$$

The above equation is like NLS equation which is un-avoided throughout modern science since it represents one of the simplest equations in which the combination of dispersive effects with nonlinearity leads to soliton solutions. Our next task is to solve the DNLS equation with the help of Hirota bilinearization method.

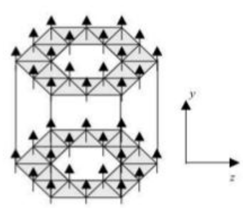

Figure 1. The arrangement of spins in a ferromagnetic nanotube.

3. One Soliton Solution

In non-linear science, many physical phenomena such as plasma physics, nonlinear optics and fluid dynamics are often related to non-linear partial differential equations (PDEs). Solutions of such nonlinear PDEs are often investigated by researchers. Many effective methods have been proposed and developed, such as the inverse scattering method, Wronskian technique, Hirota's Bilinear (HB) method, tanh method, Backlund transformation, Darboux transformation, Jacobi elliptic function expansion method, Painleve expansion, Fan sub equation method, homogeneous balance method, exponential function method and Subsidiary equation method [11-12]. However, HB method is widely used to construct the multi-soliton solutions of many nonlinear PDEs. There is no general rule for such method of bilinear form and one can try to take some transformation like logarithmic transformation or rational transformation. Till the present research, HB method is extended uniformly to all the nonlinear PPEs contained in the iso-spectral AKNS hierarchy, the modified Kdv(mKdv) hierarchy and the variable coefficient KdV hierarchy.

Therefore, in order to solve the Eq. (2) we use bilinearization transformation.

$$s_i^+ = \frac{g_n}{f_n} \tag{3}$$

whereg_n is a complex function and f_n is the real function of x and t. Substituting the above transformation in Eq. (2),

we arrived

$$\begin{gathered} A_1(g_n . f_n) = 0, A_2(f_n . f_n) = g_n g_n^*, A_3(g_n f_n) g_n g_n^* = 0, \\ A_4(f_n f_n . g_n g_n^*) = g_n^2 g_n^{*^2}, A_5 g_n^5 g_n^{*^4} = 0, \end{gathered} \tag{4}$$

where,

$$A_1 = iD_t + \frac{Ja^2}{g\mu_B}[coshD_n - 1] - 2D_1 - 4D_2 - \frac{\eta g \mu_B H_0}{a^3}$$

$$A_2 = \frac{Ja^2}{g\mu_B(D_1+D_2)}[coshD_n - 1]$$

$$A_3 = -\frac{1}{2}\frac{Ja^2}{g\mu_B} coshD_n$$

$$A_4 = \frac{1}{6}\frac{Ja^2}{g\mu_B D_2}[cosh2D_n]$$

$$A_5 = -\frac{D_2}{8}.$$

Where the HB operators D_n and D_t are defined by

$$D_n^n D_t^m (a.b) = \left(\frac{\partial}{\partial n} - \frac{\partial}{\partial \acute{n}}\right)^n \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial \acute{t}}\right)^m a(n,t) b(\acute{n},\acute{t})|_{n=\acute{x}, t=\acute{t}}. \tag{5}$$

The above set of equations can be solved by introducing the following power series expansions for g_n and f_n

$$g_n = \epsilon g_n^{(1)}$$

$$f_n = 1 + \epsilon^2 f_n^{(2)} \tag{6}$$

Where ϵ is the formal expansion parameter. The resulting set of equations, after collecting the terms with the same power in ϵ recasts as follows

$$\epsilon^1 : A_1 g_n^{(1)} = 0,$$
$$\epsilon^2 : A_2 \left[f_n^{(2)} + f_n^{(2)} \right] + A_4 \left[g_n^{(1)} g_n^{(1)^*} \right] = g_n^{(1)} g_n^{(1)^*},$$
$$\epsilon^3 : A_1 \left[g_n^{(1)} f_n^{(2)} \right] + A_3 \left[g_n^{(1)^2} + g_n^{(1)^*} \right],$$
$$\epsilon^4 : A_2 \left[f_n^{(2)} f_n^{(2)} \right] + A_4 \left[g_n^{(1)} g_n^{(1)^*} f_n^{(2)} + g_n^{(1)} g_n^{(1)^*} f_n^{(2)} \right] = g_n^{(1)^2} \left(g_n^{(1)^*} \right)^2,$$
$$\epsilon^5 : A_3 \left[g_n^{(1)^2} g_n^{(1)^*} f_n^{(2)} \right] = 0,$$
$$\epsilon^6 : g_n^{(1)^5} \left(g^{(1)^*} \right)^4 = 0. \qquad \textbf{(7)}$$

We assume

$$g_n^{(1)} = e^{\theta_1}, f_n^{(2)} = e^{\theta_1 + \theta_1^* + \eta}, \theta_1 = kn + \omega t + \theta_1^0 \qquad \textbf{(8)}$$

Using above solution, we solve Eq. (7) and obtain

$$\omega = i \left[\frac{Ja^2}{g\mu_B} [coshk - 1] + 2D_1 + 4D_2 - \frac{\eta g \mu_B H_0}{a^3} \right],$$
$$e^{\eta} = -\frac{D_1 + D_2}{24D_2} \left[sinh^{-2} \left(\frac{k+k^*}{2} \right) + 2sinh^{-2} \left(\frac{k+k^*}{2} \right) sinh^2 (k + k^*) \right]. \qquad \textbf{(9)}$$

Using the Eq.(9), we can write the one soliton solution of Eq.(2),

$$S_i^+ = \frac{sinh^2\left(\frac{k+k^*}{2}\right) cosh^{-1}(k+k^*) exp\left\{ kn + i\left[\frac{Ja^2}{g\mu_B}[coshk-1] + 2D_1 + 4D_2 - \frac{\eta g \mu_B H_0}{a^3} \right] \right\}}{sinh^2\left(\frac{k+k^*}{2}\right) cosh^{-1}(k+k^*) - \frac{D_1+D_2}{24D_2} e^{(k+k^*)n + 2\theta_1^0}}. \qquad \textbf{(10)}$$

4. Results and Discussion

Equation (10) was plotted with choice of parameters $D_1 = 0.4$, $g = 0.1$, $\mu = 6.2$, $H = 0.01$and $r = 1$. For first case, exchange interaction parameter $J=0.01$ to 0.8 with low value of higher order single ion anisotropyD_2=0.1 were varied as shown in Fig.(2a and 1b). It was observed that a bell shape soliton with amplitude as 0.3 with J value as 0.01 as shown in Fig (2a). When J as 0.8, obtained the low amplitude soliton and the shape of solitonas changed as depicted in Fig.(2b). It was noticed that soliton amplitude was very low for lower value of higher order single ion anisotropy. For the second case, higher energetic soliton pulses were obtained for high value of single ion anisotropy as shown in Fig.(3a and 3b). From Fig. (3a), we observed that when J as 0.01 which induced the high amplitude soliton with small spikes generated on the top of the soliton propagating along FN spin system.

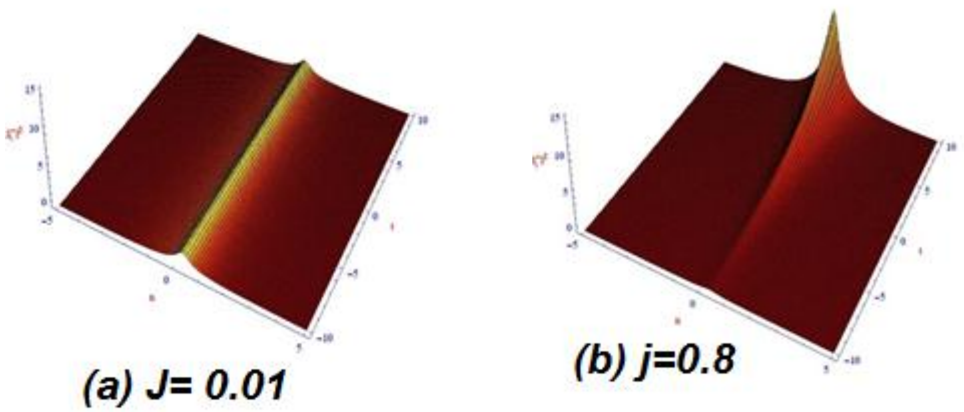

Figure 2. Snapshots of one soliton solution with choice of parameters D_1= 0.4, g= 0.1, µ= 6.2, H= 0.01, r= 1

Similar [Fig.(3b)] energetic soliton pulse was obtained for the value of *J=0.8*. Therefore, the strength of higher order single ion anisotropy induces the high amplitude or high energetic soliton wave in FN. As a result, single ion anisotropy and spin exchange interaction is significant factor for inducing high energetic soliton waves in FN system.

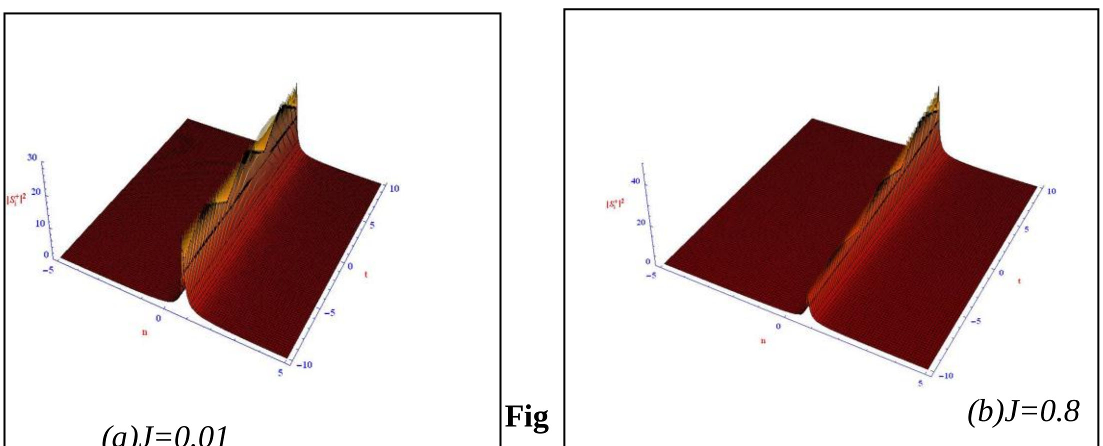

Figure 3 (a and b). Snapshots of one soliton solution with choice of parameters D_2= 1.0

5. Conclusion

In this paper, we studied the dynamics of soliton in classical discrete Heisenberg FN. With the help of classical approach, we obtained the DNLS equation which associates the dynamics of spins in FN. One soliton solution of DNLS equation has been obtained by solving DNLS equation with aid of HB method. The obtained one soliton solution describes the dynamics of SW in FN system. As per the graphical results, we strongly believed that exchange interaction and single ion anisotropy parameter affect the evolution of soliton along FN.

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