Flow and Heat Transfer in Bingham-Papanastasiou Fluid over a Wedge

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Abstract

This investigation focuses on flow and corresponding heat transfer behavior of viscoplastic Bingham-Papanastasiou fluid over a static radiated wedge. The modeled boundary layer flow governing equations are simplified by using similarity transformations. Numerical results are obtained for the economized equations by employing Runge-Kutta and Newton's method. The impact of the associated parameters concerned with the fluid properties, friction factor and local Nusselt number are determined. It is found that the thermal radiation parameter improves the heat transfer rate. Bingham number boosts the friction at the wall and reduces the heat transfer rate. Improvement in Bingham number reduces the free surface movement and velocity of the fluid increases.

Keywords: *Bingham model, Bingham-Papanastasiou model, Static Wedge, Yield-stress, non-Newtonian dissipation, thermal radiation.*

1. Introduction.

Viscoplastic materials like slurries, paints, mud, cement, pastes, radioactive nuclear waste and food products such as margarine, ketchup and mayonnaise often encountered in industrial problems exhibits yield stress, a significant value of stress beneath which they do not flow. Barnes [1] discussed the yield-stress fluid behavior, though the theory of true yield stress remains divisive subject, an apparent yield stress supposed to be applicable for engineering purposes. Recently Bird et al. [2] listed several materials which exhibits yield stress. Viscoplastic models include Bingham plastic, nonlinear Casson model, Herschel-Bulkley model, with power-law viscous dependency. Among all these discontinuous models, Bingham plastic model proposed by Bingham [3] is the simplest as well as most employed one in the industrial applications. Viscoplastic Bingham model exhibit merely two rheological parameters,(i) a yield stress limit (ahead of which the material flow similar to Newtonian fluid) (ii) a constant viscosity (being characterized with the existence of a yield surface which separates yielded and unyielded regions).

The conventional Bingham model does not yield any useful information regarding the stress distribution when the extra stress is smaller than the yield stress, identifying this material rigidness Papanastasiou [4] proposed a constitutive relation - an exponential function controlled with a non-rheological parameter m, exponential function assures that even for the extremely minute value of the yield stress the shear rate viscosity stay finite. This new model, Bingham-Papanastasiou's model proclaims the original stress discontinuous behavior of Bingham model and is valid for all rates of deformation and is convenient to implement. Ellwood et al. [5] considered the Papanastasiou's model to analyze the steady and transient performance of jets generated by circular and slit nozzles, and found that yield stress suppresses contraction of the jet. Recently Mitsoulis et al.[6],Mitsoulis[7],Belblidia [8] ,Rees et al. [9] and Nadeem et.al [10] studied viscoplastic and viscoelastoplastic fluids.

Falkner and Skan [11] analyzed the steady laminar flow past a static wedge to exemplify the applications of Prandtl's boundary layer theory. In their study, by employing similarity transformation, the Prandtl's boundary layer equations were reduced to a nonlinear ordinary differential equation; it is well-known as

Falkner-Skan equations. Hartree [12] studied the explanations and dependence on pressure gradient β .The flow over a wedge has various industrial applications such as metal spinning, plastic films, polymer extrusion and metallurgical processes hence here are several number of literature [13-20] available on Falkner-Skan flow in view of diverse parameters effects.

The consequence of thermal radiation on the convective heat and mass transport problems has been studied by numerous researchers owing to its relevance in physics and engineering. Radiation effects play a vital role in cooling of nuclear reactor, polymer processing industry, high temperature plasmas, power generation systems, liquid metal fluids, magnetohydrodynamic accelerators etc. The influence of radiation is essential when there is a large dissimilarity in the surface along with the ambient temperature. Thermal radiation heat transfer phenomenon is clearly presented in text by Sparrow and Cess [21], Howell [22]. Raju et al. [23] investigated radiation and aligned magnetic field on the flow of ferrofluids by considering non-uniform heat/sink. Hayat et al. [24] considered Joule heating and solar radiation in convective flow of MHD thixotropic nanofluid. Ara et al. [25] studied radiation influence on Eyeing-Powell fluid past an exponentially shrinking sheet. Sheikholeslami et al. [26] considered thermal radiation of ferrofluid with variable viscosity and Lorentz forces.

Our principal focus in the present study is to analyze momentum and thermal behavior of viscoplastic Bingham-Papanastasiou fluid over a static Wedge with the influence of thermal radiation.

2. Mathematical formulation.

Consider a steady, incompressible two dimensional inelastic viscoplastic Bingham-Papanastasiou fluid over a static wedge with the influence of thermal radiation. The free stream velocity is $u_e(x)$. x - coordinate measured all along the surface of the wedge. The total angle of the wedge is defined to be $\Omega = \lambda\pi$. Here $\lambda = \frac{2n}{n+1}$ represents wedge angle parameter. The wedge surface is maintained with the variable temperature $T_w(x)$. T_∞ is the ambient temperature. In Fig.1 the physical flow configuration and coordinate system are presented. The rheological model of Bingham-Papanastasiou plastic fluid is given as :(see Belblidia et al.[8])

$$\tau_{ij} = \mu_p + \frac{\tau_y}{\dot{\gamma}}\left(1 - e^{-m\dot{\gamma}}\right)A \qquad \text{for } \tau \geq \tau_y \tag{1}$$

$$A = 0 \text{ for } \tau \prec \tau_y$$

Where $\tau_{ij}, \mu_p, \tau_y, A = \nabla\Delta + (\nabla\Delta)^t, n, m$ represents deviatoric stress tensor, plastic viscosity, yield stress, rate of strain tensor, ∇ the differential operator , V the velocity (u, v) vector defined in the Cartesian coordinates. n the power-law index parameter, m the stress growth exponent.

The second invariant of rate of strain tensor $\dot{\gamma}$ is defined as $\dot{\gamma} = \sqrt{\frac{1}{2} tr\, A^2}$ (2)

With the above considered assumptions, the flow governing equations are :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho}\left(\mu + m\tau_y e^{-m\frac{\partial u}{\partial y}}\right)\frac{\partial^2 u}{\partial y^2} \tag{4}$$

$$\rho C_P\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\tau_y\left(1-e^{-m\frac{\partial u}{\partial y}}\right)\frac{\partial u}{\partial y}+\frac{16\sigma_1 T_\infty^3}{3k_1}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

The corresponding boundary condition for the model is:

$$at\ y=0,\quad u=0,\quad v=0,\quad T=T_w(x)$$

$$\text{as}\ y\to\infty,\quad u\to u_e(x),\quad T=T_\infty \tag{6}$$

$$u_e(x)=ax^n,\quad T_w(x)=T_\infty+cx^n \tag{7}$$

To conquer similarity solution to the Eqs.(3) to(5) with the boundary conditions (6) the following similarity transformations and the dimensionless variables are employed.

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

$$\psi=\sqrt{x\nu u_e(x)}\,f(\zeta),\quad T=T_\infty+(T_w-T_\infty)\theta(\zeta),\quad \zeta=\sqrt{\frac{u_e(x)}{x\nu}}y \tag{8}$$

From the above considered transformations, the non-dimensional, nonlinear coupled set of ordinary differential equations (ODEs) is obtained as:

$$\left(1+mBne^{-m\sqrt{\mathrm{Re}}f''}\right)f'''+n\left(1-2f'^2\right)+\frac{n+1}{2}ff''=0 \tag{9}$$

$$\left(1+\frac{4}{3}R\right)\theta''+\Pr\left(\frac{n+1}{2}f\theta'-nf'\theta\right)+\Pr Ec\left(\frac{Bn}{\sqrt{\mathrm{Re}}}\left(1-e^{-m\sqrt{\mathrm{Re}}f''}\right)f''+f''^2\right)=0 \tag{10}$$

The transformed boundary conditions are:

$$f(0)=0,\quad f'(0)=0,\quad \theta(0)=1,\ f'(\infty)=1,\ \theta(\infty)=0 \tag{11}$$

Where, Kinematic plastic viscosity- $v_p=\dfrac{\mu_p}{\rho}$, Reynolds number- $\mathrm{Re}=\dfrac{a^3x^{3n-1}}{\nu}$, Bingham number- $Bn=\dfrac{\tau_y}{\mu_p}$, Prandtl number- $\Pr=\dfrac{\mu C_P}{k}$, Eckert number- $Ec=\dfrac{\left(ax^n\right)^2}{C_P\left(T_w-T_\infty\right)}$,Thermal radiation parameter- $R=\dfrac{4\sigma_1 T_\infty^3}{k_1 k}$

Physical quantities local skin-friction coefficient C_f and the local Nusselt number N_u are defined as:

Local skin-friction coefficient

$$C_f=\left.\frac{\tau_{xy}}{\mu}\right|_{y=0}$$

$$C_f=\sqrt{\mathrm{Re}}\left(f''(0)+\frac{Bn}{\sqrt{\mathrm{Re}}}\left(1-e^{-m\sqrt{\mathrm{Re}}f''(0)}\right)\right) \tag{12}$$

Local Nusselt number

$$N_u = -\sqrt{\mathrm{Re}}\left(1+\frac{4}{3}R\right)\theta'\ (0) \tag{13}$$

3. Results and discussion

The non-dimensional, nonlinear, coupled Eqs.(9)and (10) with the boundary conditions (11) are solved numerically with the application of Runge-Kutta and Newton's methods. In order to simulate the problem the values of non-dimensional parameters are considered as $n = 0.3, m = 0.1, \mathrm{Re} = 10, \mathrm{Pr} = 2, Ec = 0.5, Bn = 2, R = 0.5$ .These values are kept constant besides the varied variables as described in tables and figures.

Fig.2. shows the influence on temperature distribution for increasing values of Eckert number, Ec .Here we observe that increasing values of Ec , significantly increases the temperature distribution in the flow region. In Fig.3 we observe that increment in radiation parameter, R , decreases the temperature distribution in the flow region. Generally rise in thermal radiation generates heat energy in the flow, whereas due to the dominance of dissipation we have seen decrement in the temperature field. Fig.4. demonstrates the influence of Prandtl number, Pr on the temperature distribution. Temperature profile gradually declines with the increment in Pr .Since higher Prandtl number reduces conduction as a result the thermal boundary layer thickness reduces. Fig.5 and Fig.6 illustrates the influence of Power law index, n on the velocity and temperature distribution. Increasing values of n improves both momentum and thermal boundary thickness, the increasing values of power law index improves the pressure on the flow, these forces helps to improve the both thermal and momentum profiles.

Fig.7 and Fig.8 portrays that increment in Bingham number, Bn , reduces the free surface movement hence there is decrement in momentum boundary layer and viscoplasticity serves to flatten the free surface profiles hence there is increment in temperature distribution. Fig.9 and Fig.10 is plotted to observe the variation in momentum and thermal boundary layer of Bingham-Papanastasiou fluid with the increment in Reynolds number, Re .Increment in Re improves the momentum boundary layer and decreases the temperature distribution. In Fig.11 and Fig.12 it is observed that improvement in stress growth exponent m improves temperature distribution and decelerates velocity boundary thickness this is due to improvement in m inhibits the stress growth.

Table I exhibit the comparative study of present analysis with the established report of Frank [27] and are in excellent agreement.

Table II. illustrates the variation of Skin friction coefficient $f''(0)$ and Nusselt number $\left(-\theta'(0)\right)$ for different values of the parameters. It is observed that Increment in Eckert number, radiation parameter, Prandtl number has no influence on skin friction coefficient. Whereas improvement in Eckert number, Power law index, Bingham number improves heat transfer performance. And improvement in radiation parameter, Prandtl number and Reynolds number lowers the heat transfer rate. Improvement in Power law index improves friction factor whereas improvement in Reynolds number, decreases the friction factor. As expected, higher values of Bingham number, initially decreases the friction factor but larger values of Bn improves the friction factor.

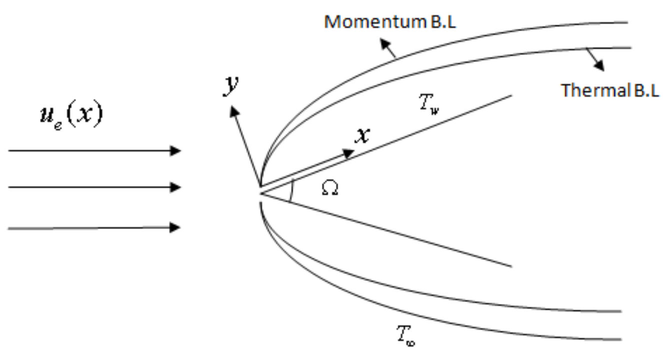

Figure1. Flow configuration and coordinate system.

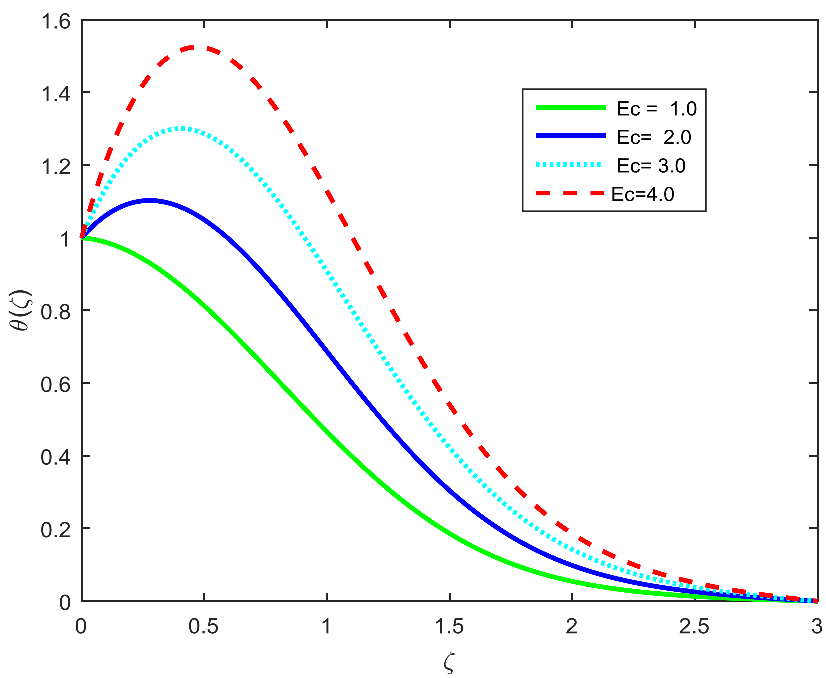

Figure 2. Influence of Ec on temperature field

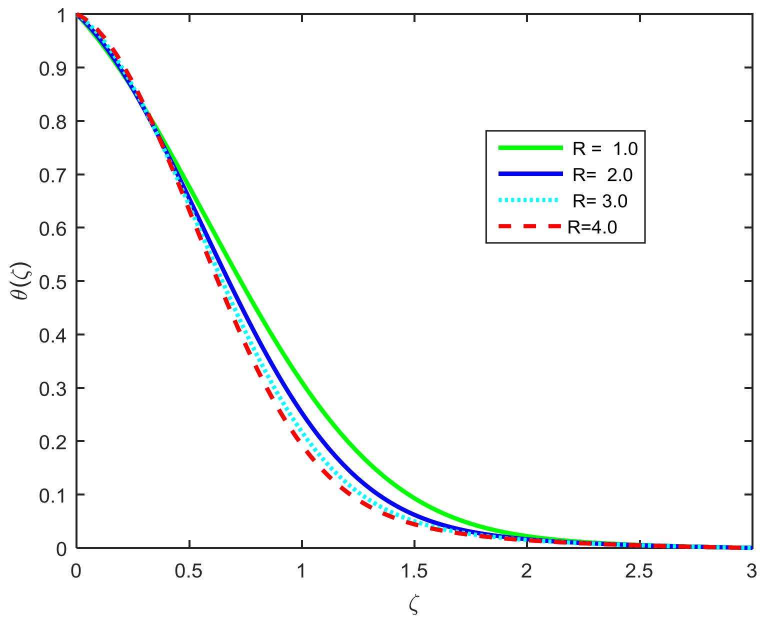

Figure 3. Influence of R on temperature field

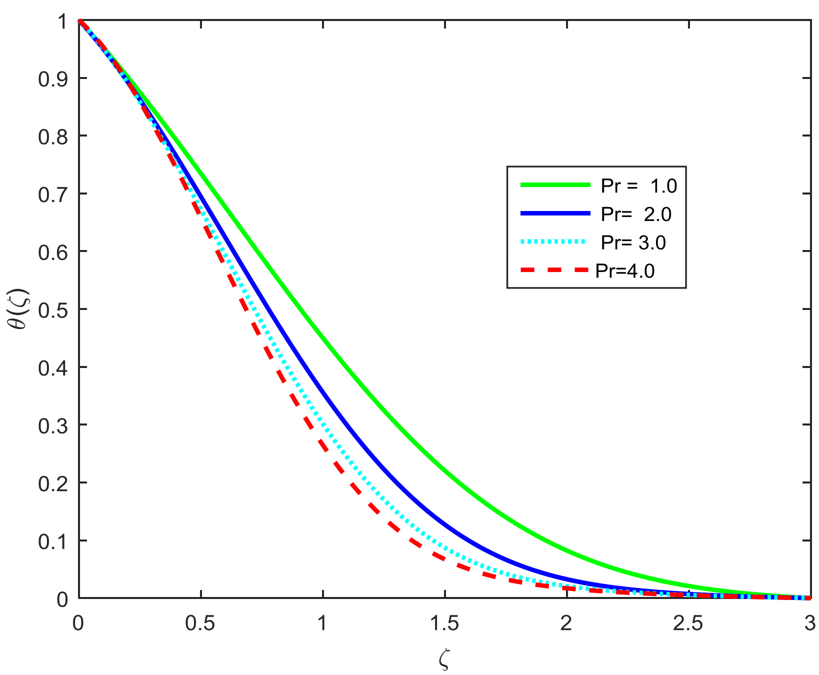

Figure 4. Influence of Pr on temperature field

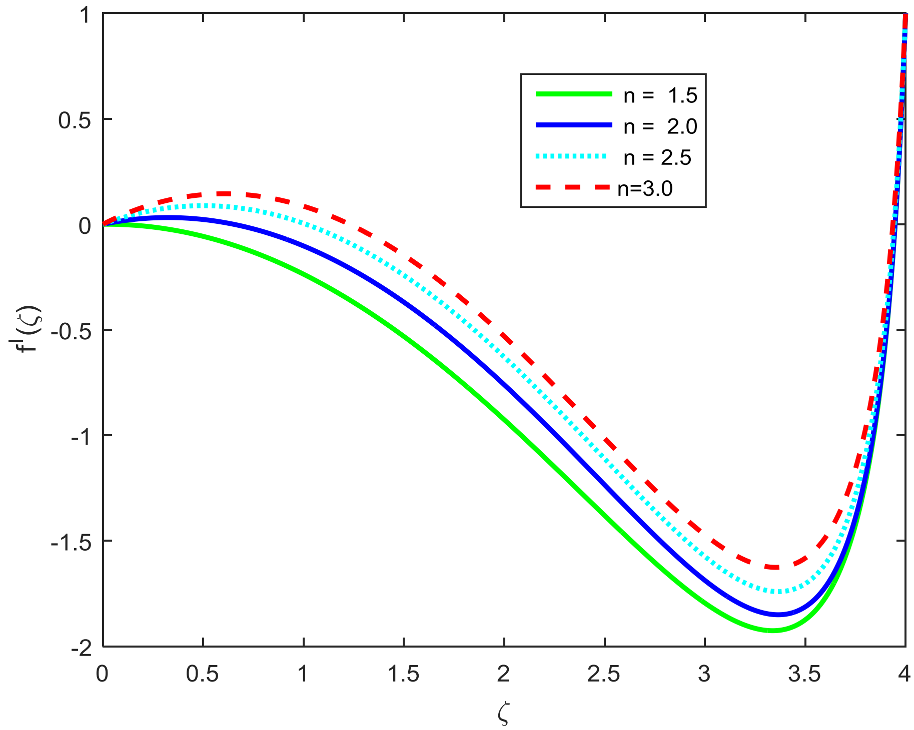

Figure 5. Influence of n on velocity field

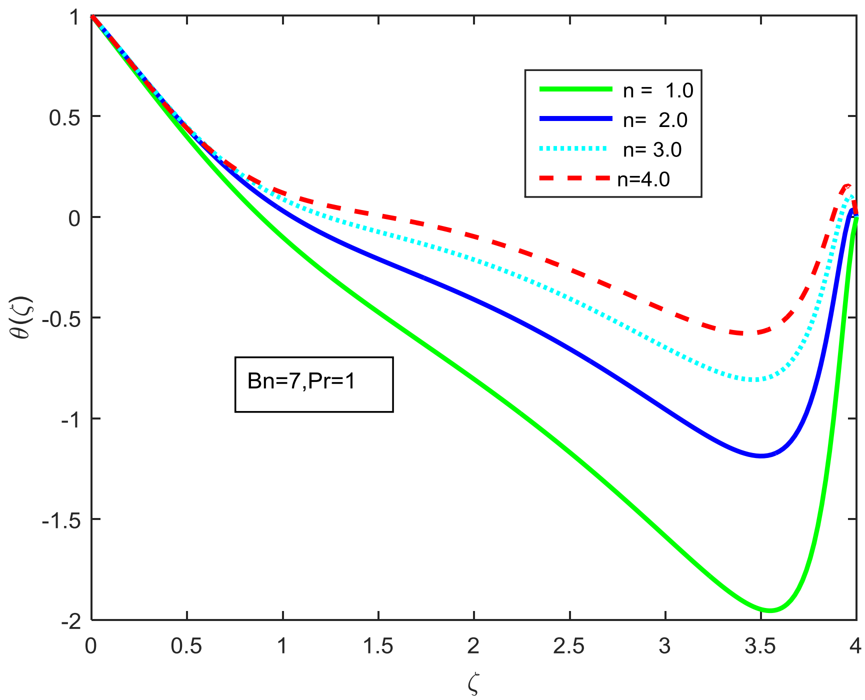

Figure 6. Influence of n on temperature field

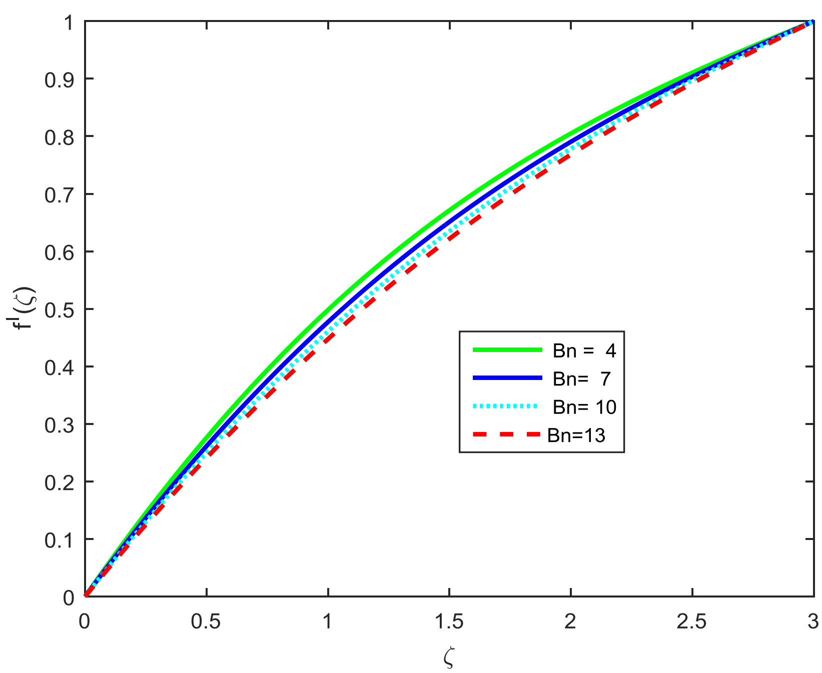

Figure 7. Influence of Bn on velocity field

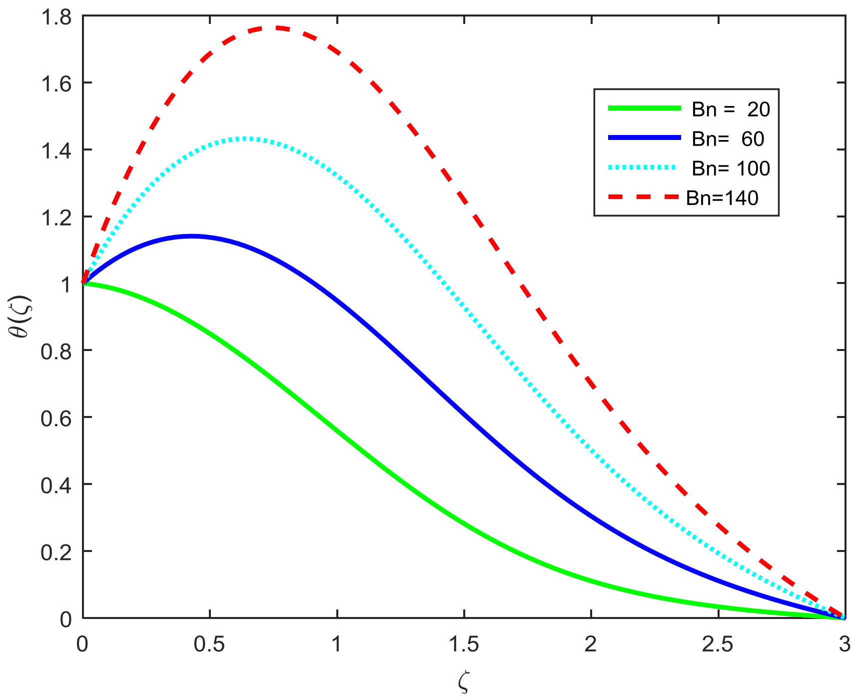

Figure 8. Influence of Bn on temperature field

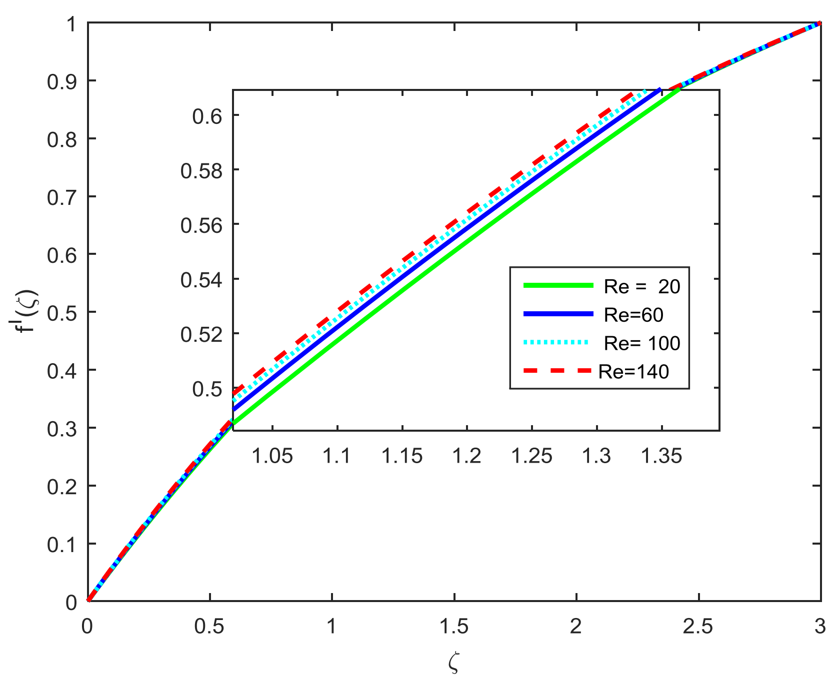

Figure 9. Influence of Re on velocity field

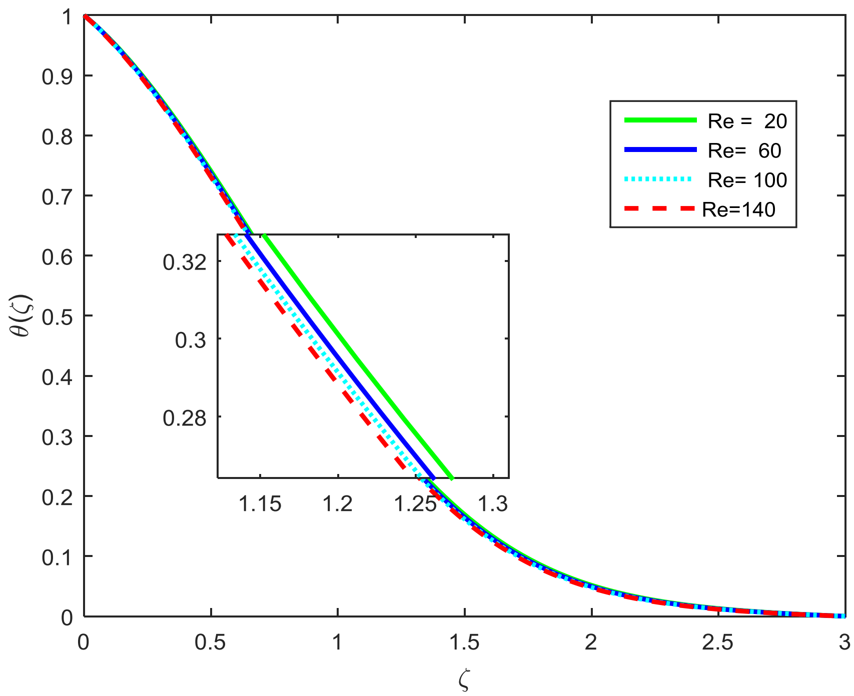

Figure 10. Influence of Re on temperature field

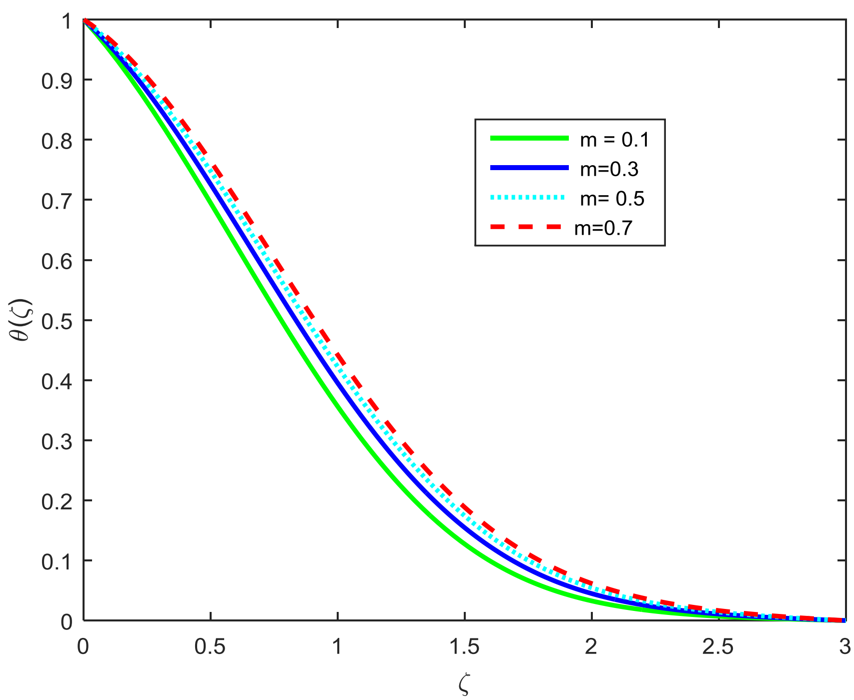

Figure 11. Influence of m on temperature field

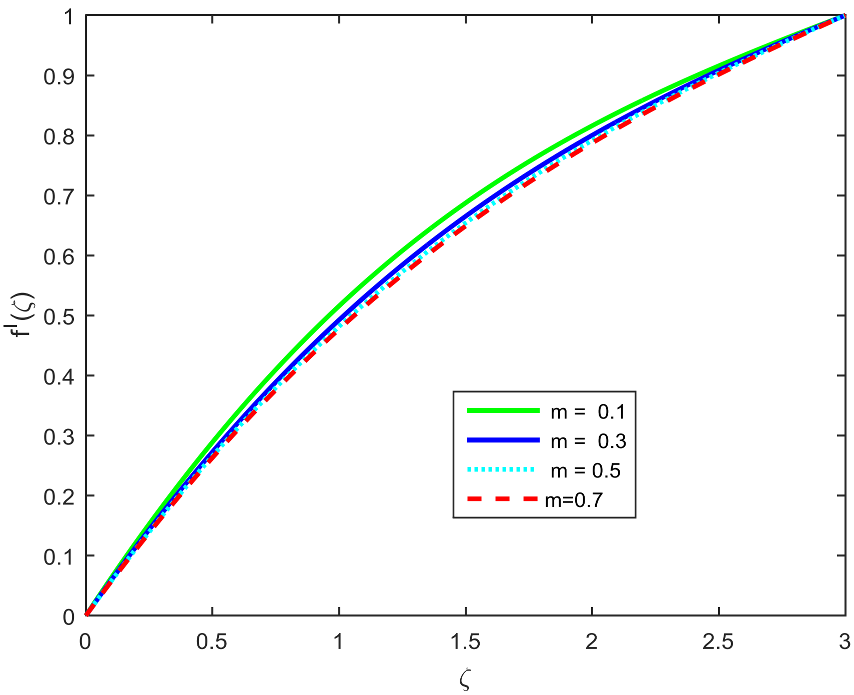

Figure 12. Influence of m on velocity field

Table1 .Comparison of the results for $f''(0)$ when $Bn = m = \Pr = R = Ec = 0$

n	Frank [27]	Present Study
0	0.46960	0.4696
1	1.23259	1.23259

Table II. Variation of Skin friction coefficient $f''(0)$ and Nusselt number $(-\theta'(0))$ for different values of the parameters

Ec	R	Pr	n	Bn	Re	$C_f\sqrt{\mathrm{Re}}$	$\left(-N_u\sqrt{\mathrm{Re}}\right)$
1.0						0.758166	0.081471
2.0						0.758166	-1.299905
3.0						0.758166	-2.681281
4.0						0.758166	-4.062658
	1.0					0.758166	1.029644
	2.0					0.758166	1.352226
	3.0					0.758166	1.396510
	4.0					0.758166	1.153879
		1.0				0.758166	0.760966
		2.0				0.758166	0.772159
		3.0				0.758166	0.722999
		4.0				0.758166	0.648520
			1.5			0.531562	3.808333
			2.0			0.964604	2.773002
			2.5			1.268081	2.573400
			3.0			1.506497	2.513137
				20		1.369393	0.116248
				60		2.654133	-1.137728
				100		3.925019	-2.351898
				140		5.193020	-3.557313

					20	0.928884	0.577208
					60	0.914222	0.593308
					100	0.904722	0.603826
					140	0.897348	0.612039

4. Conclusion

In this study, the flow and heat transfer characteristics of Bingham Papanastasiou plastic flui over a wedge is studied in the presence of thermal radiation and magnetic effects. The most significant parameters studied in this investigation were the Bingham number, Eckert number,Prandtl number, Reynolds number and radiation. The influence of pertinent physical parameters have been studied on velocity, temperature profiles, skin friction and heat transfer rate. The velocities were increased with increment in power law index. Improvement in Bingham number reduces the free surface movement and velocity of the fluid increases. Improvement in radiation has decreased the temperature of the fluid at the boundary. Improvement in Eckert number increased the heat transfer rate. Improvement in Prandtl number reduces thermal boundary thickness.

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