Solving Neutrosophic Multi-Objective Linear Fractional Programming Problem Using Central Measures – II

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Abstract

In this paper, we introduce a new transformation method for resolving the issue of Neutrosophic Multi-Objective Linear Fractional Programming Problem (NMOLFPP). Our approach involves converting NMOLFPP into a Neutrosophic Single Objective Linear Fractional Programming Problem (NSOLFPP) and proposing an algorithm to discover its solution. To strengthen our method, we present a numerical example. The outcomes of our study indicate that the Advanced Harmonic Average technique provides the best optimal solution compared to the other methods such as Arithmetic Average, New Arithmetic Average, and Harmonic Average (H_{av}) Techniques.

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Keywords: *Neutrosophic Multi-Objective Linear Fractional Programming Problem, Neutrosophic Triangular Number, Harmonic averaging techniques, and Advanced Harmonic averaging techniques.*

1. Introduction

We have developed a method to solve Neutrosophic MOLFPP and put forth an algorithm for solving it. Our approach is applicable to any number of objectives and utilizes more efficient techniques to achieve the optimal solution. To illustrate the effectiveness of our algorithm, we have included a numerical example and compared the results with other methods.

2. Preliminaries

In order to develop our main point, we must recall some definitions and results.

Definition:2.1 (Single Valued Triangular Neutrosophic Number (SVTNN)). A SVTNN is defined by $\tilde{A}^* = \{(a_1^l, b_1^m, c_1^u); \tau_a, i_a, \omega_a\}$ whose three membership functions for the truth, indeterminacy, and a falsity of X are given by

$$\tau_{\tilde{A}^*}(x) = \begin{cases} \frac{(x-a_1^l)\tau_a}{b_1^m - a_1^l} & (a_1^l \le x < b_1^m) \\ \tau_a & (x = b_1) \\ \frac{(c_1^u - x)\tau_a}{c_1^u - b_1^m} & (b_1^m \le x < c_1^u) \\ 0 & Otherwise, \end{cases}$$

$$i_{\tilde{A}^*}(x) = \begin{cases} \frac{(b_1^m - x)i_a}{b_1^m - a_1^l} & (a_1^l \le x < b_1^m) \\ i_a & (x = b_1^m) \\ \frac{(x - c_1^u)i_a}{c_1^u - b_1^m} & (b_1^m \le x < c_1^u) \\ 1 & Otherwise, \end{cases}$$

$$\omega_{\tilde{A}^*}(x) = \begin{cases} \frac{(b_1^m - x)\omega_a}{b_1^m - a_1^l} & (a_1^l \le x < b_1^m) \\ \omega_a & (x = b_1^m) \\ \frac{(x - c_1^u)\omega_a}{c_1^u - b_1^m} & (b_1^m \le x < c_1^u) \\ 1 & Otherwise, \end{cases}$$

where $0 \le \tau_{\tilde{A}^*}(x) + i_{\tilde{A}^*}(x) + \omega_{\tilde{A}^*}(x) \le 3,\ x \in \tilde{A}^*$. Additionally, when $a_1^l > 0,\ \tilde{A}^*$ is called a positive SVTNN. Similarly, when $a_1^l < 0,\ \tilde{A}^*$ becomes a negative SVTNN.

Definition:2.2. Let $\tilde{a}^n = \{(a_1^l, b_1^m, c_1^u); \tau_a, i_a, \omega_a\}$ and

$\tilde{b}^n = \{(a_2^l, b_2^m, c_2^u); \tau_b, i_b, \omega_b\}$ be two SVTNN's and $\gamma \ne 0$. Then

a) *Addition:*

$$\tilde{a}^n + \tilde{b}^n = \{(a_1^l + a_2^l, b_1^m + b_2^m, c_1^u + c_2^u); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}$$

b) *Subtraction:*

$$\tilde{a}^n - \tilde{b}^n = \{(a_1^l - c_2^u, b_1^m - b_2^m, c_1^u - a_2^l); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}$$

c) *Multiplication:*

$$\tilde{a}^n . \tilde{b}^n = \begin{Bmatrix} Min(a_1^l a_2^l, a_1^l c_2^u, c_1^u a_2^l, c_1^u c_2^u), b_1^m b_2^m, Max\,(a_1^l a_2^l, a_1^l c_2^u, c_1^u a_2^l, c_1^u c_2^u); \\ \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b \end{Bmatrix}$$

d) *Division:*

$$\frac{\tilde{a}^n}{\tilde{b}^n} = \left\{ Min\left(\frac{a_1^l}{a_2^l}, \frac{a_1^l}{c_2^u}, \frac{c_1^u}{a_2^l}, \frac{c_1^u}{c_2^u}\right), \frac{b_1^m}{b_2^m}, Max\left(\frac{a_1^l}{a_2^l}, \frac{a_1^l}{c_2^u}, \frac{c_1^u}{a_2^l}, \frac{c_1^u}{c_2^u}\right); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b \right\}$$

e) *Scalar Multiplication:*

$$\gamma \tilde{a}^n = \begin{cases} \{(\gamma a_1^l, \gamma b_1^m, \gamma c_1^u); \tau_a, i_a, \omega_a\}, (\gamma > 0) \\ \{(\gamma c_1^u, \gamma b_1^m, \gamma a_1^l); \tau_a, i_a, \omega_a\}, (\gamma < 0). \end{cases}$$

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3. Molfpp In Neutrosophic Environment

The ratio objective function that has numerator and denominator, and is defined as follows:

$$Max\ \tilde{Z}_i \text{ and } Min\ \tilde{Z}_i = \frac{\tilde{C}_i \tilde{x}^T + \tilde{\alpha}_i}{\tilde{D}_i(\tilde{x}^T + \tilde{1})} \quad \forall \begin{cases} Max\ \tilde{Z}_i & if\ i = 1,2,\dots,r \\ Min\ \tilde{Z}_i & if\ i = r+1, r+2,\dots,s \end{cases} \tag{1}$$

subject to $\quad \tilde{A}\tilde{x} \le \tilde{B}$ (2)

$\tilde{x} \ge \{(0,0,0); 1,0,0\}$ (3)

where $\tilde{x}^T$ and $\tilde{x}$ is a n-dimensional vector of decision variables
r is the number of objective functions that is to be maximized
$s-r$ is the number of objective functions that is to be minimized
$\tilde{C}_i$ and $\tilde{D}_i (\forall i = 1,2,\dots,r,r+1,\dots,s)$are n-dimensional vector of SVTNN.
$\tilde{A}$ is a $m \times n$-matrix of co-efficient (SVTNN).
$\tilde{\alpha}_i$ is a neutrosophic constant.
$\tilde{1} = \{(1,1,1); 1,0,0\}$

4.Different Kind Of Techniques

Objectives are the specific goals that need to be met in order to optimize something. In this case, the objective functions are being optimized individually, subject to the constraints (2) and (3).

$$Max\ \tilde{Z}_i \text{ and } Min\ \tilde{Z}_i = \tilde{\varphi}_i \qquad \forall \begin{cases} Max\ \tilde{Z}_i & if\ i = 1,2,\dots,r \\ Min\ \tilde{Z}_i & if\ i = r+1, r+2,\dots,s \end{cases} \tag{4}$$

where $\tilde{\varphi}_1^{Max}$, $\tilde{\varphi}_2^{Max}, \dots, \tilde{\varphi}_r^{Max}$, $\tilde{\varphi}_{r+1}^{Min}, \dots, \tilde{\varphi}_s^{Min}$ are the optimal values of the objective functions.

It can be seen in the problem that the following notations are used.

$\tilde{\varphi}_i^{Max}$ = a value to be maximized for each objective function.

$\tilde{\varphi}_i^{Min}$ = a value to be minimized for each objective function.

$$|\tilde{Z}_i^{Max}| = \tilde{\varphi}_i^{Max}; \quad and \quad |\tilde{Z}_i^{Min}| = \tilde{\varphi}_i^{Min}\ ;\ SM = \sum_{i=1}^{r} \frac{\tilde{Z}_i}{\tilde{\varphi}_i^{Max}} \quad and \quad SN = \sum_{i=r+1}^{s} \frac{\tilde{Z}_i}{\tilde{\varphi}_i^{Min}}$$

- Arithmetic Average Technique

$$\tilde{m}_1 = \min(\varphi_i^{Max})\ ; \quad \tilde{m}_2 = \min(\varphi_i^{Min})\ ; \quad AV_2 = \frac{\tilde{m}_1 + \tilde{m}_2}{2}$$

$$Max\ Z = \frac{SM - SN}{AV_2} \tag{5}$$

- New Arithmetic Average Technique

$$\tilde{m}_1 = \min(\varphi_i^{Max})\ ; \quad \tilde{m}_2 = \min(\varphi_i^{Min})\ ; \quad AV_s = \frac{\tilde{m}_1 + \tilde{m}_2}{s}$$

$$Max\ Z = \frac{SM - SN}{AV_s} \tag{6}$$

- Harmonic Average Technique

$$H_{av_1} = H_{av}(\tilde{\varphi}_i^{Max}); \quad H_{av_2} = H_{av}(\tilde{\varphi}_i^{Min}); \quad S_1 = \frac{SM}{H_{av_1}}; \quad S_2 = \frac{SN}{H_{av_2}}$$

$$Max\ Z = S_1 - S_2 \tag{7}$$

- Advanced Harmonic Average Technique

$$m_1 = \min\left(\left|\varphi_i^{Max}\right|\right); \quad m_2 = \max\left(\left|\varphi_i^{Min}\right|\right); \quad AH_{av} = \frac{2|m_1||m_2|}{|m_1| + |m_2|}$$

$$Max\ Z = \frac{SM - SN}{AH_{av}} \quad (8)$$

4.1 Procedure For Determining Of Combined Single Objective Function

The following procedure is adapted to obtain the solution for the Neutrosophic MOLFPP defined in previous can be summarized as follows.

Step 1. To obtain the optimal solution to the linear fractional programming problem, use the modified simplex method (using [7]) in which each objective function has to be maximized or minimized and to find the optimal solution.

Step 2. The feasibility of the solution obtained in Step 1 should be checked, if it is feasible, then the next step should be Step 3. If it is not feasible, then it is best to use dual simplex techniques to remove the infeasibility.

Step 3. There are various types of techniques you can use to derive the objective function.

- Calculate Neutrosophic SOLFPP equation (5), using $Max\ \tilde{Z} = (SM - SN)/AV_2$, where $AV_2 = \frac{m_1+m_2}{2}$ is arithmetic mean.
- Calculate Neutrosophic SOLFPP equation (6), using $Max\ \tilde{Z} = (SM - SN)/AV_s$, where $AV_s = \frac{m_1+m_2}{s}$ is new arithmetic average. And then s is the number of objective function.
- Calculate Neutrosophic SOLFPP equation (7), using $Max\ Z = S_1 - S_2$ where $S_1 = \frac{SM}{H_{av_1}}$ and $S_2 = \frac{SM}{H_{av_2}}$
- Calculate Neutrosophic SOLFPP equation (8), using $Max\ Z = \frac{SM-SN}{AH_{av}}$ where $AH_{av} = \frac{2|m_1||m_2|}{|m_1|+|m_2|}$ is advanced harmonic average technique.

Step 4. By repeating Step 1 to Step 3 in order to optimize the combined objective function under the same constraints, the same optimization procedure will be applied.

Numerical Example

Example 1. Solve the following MOLFPP in Neutrosophic environment.

$$Max\ \tilde{Z}_1 = \frac{\{(2,3,4); .4, .3, .3\}\tilde{x}_1 + \{(-3,-2,-1); .3, .5, .2\}\tilde{x}_2}{\{(1,2,3); .3, .5, .2\} \times \left(\tilde{x}_1 + \tilde{x}_2 + \tilde{1}\right)}$$

$$Max\ \tilde{Z}_2 = \frac{\{(8,9,10); .3, .5, .2\}\tilde{x}_1 + \{(2,3,4); .3, .4, .3\}\tilde{x}_2}{\{(1,2,3); .3, .5, .2\} \times \left(\tilde{x}_1 + \tilde{x}_2 + \tilde{1}\right)}$$

$$Max\ \tilde{Z}_3 = \frac{\{(2,3,4); .5, .4, .1\}\tilde{x}_1 + \{(-6,-5,-4); .4, .2, .4\}\tilde{x}_2}{\{(2,4,6); .3, .5, .2\} \times \left(\tilde{x}_1 + \tilde{x}_2 + \tilde{1}\right)}$$

$$Min\ \tilde{Z}_4 = \frac{\{(-7,-6,-5); .4, .4, .2\}\tilde{x}_1 + \{(1,2,3); .3, .3, .4\}\tilde{x}_2}{\{(2,4,6); .3, .5, .2\} \times \left(\tilde{x}_1 + \tilde{x}_2 + \tilde{1}\right)}$$

$$Min\ \tilde{Z}_5 = \frac{\{(-4,-3,-2); .3, .3, .4\}\tilde{x}_1 + \{(-2,-1,-1); .3, .4, .3\}\tilde{x}_2}{\{(1,2,3); .3, .5, .2\} \times \left(\tilde{x}_1 + \tilde{x}_2 + \tilde{1}\right)}$$

subject to

$$\{(1,1,1); .4, .3, .3\}\tilde{x}_1 + \{(1,1,2); .6, .2, .2\}\tilde{x}_2 \leq \{(1,2,3); .6, .3, .1\}$$

$$\{(8,9,10); .2, .6, .2\}\tilde{x}_1 + \{(1,1,2); .4, .4, .3\}\tilde{x}_2 \leq \{(8,9,10); .3, .4, .3\}$$

$$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0); 1,0,0\}$$

After finding the value of each individual objective functions by using Modified Simplex Technique for FLPP. And then the numerical results, and same variable co-efficient are given below

Table – 2: Results of the Example by using Modified Simplex Technique

i	ϕ_i	x_i	$\lvert Z_i^{Max}\rvert = \varphi_i^{Max}$ $\forall i = 1,2,\dots,r$	$\lvert Z_i^{Min}\rvert = \varphi_i^{Min}$ $\forall i = r+1, r+2, \dots, s.$
1	$\{(0.23, 0.75, 2.77); .2, .6.3\}$	$\{(0.8, 1, 1.25); .2, .6, .3\}$, $\{(0, 0,0); .2, .6, .3\}$	$\{(0.23, 0.75, 2.77); .2, .6.3\}$	$-$
2	$\{(0.94, 2.25, 6.94); .2, .6, .3\}$	$\{(0.8, 1, 1.25); .2, .6, .3\}$, $\{(0, 0,0); .2, .6, .3\}$	$\{(0.94, 2.25, 6.94); .2, .6, .3\}$	$-$
3	$\{(0.11, 0.37, 1.38); .2, .6, .3\}$	$\{(0.8, 1, 1.25); .2, .6, .3\}$, $\{(0, 0,0); .2, .6, .3\}$	$\{(0.11, 0.37, 1.38); .2, .6, .3\}$	$-$
4	$\{(0.29, 0.75, 2.43); .2, .6, .3\}$	$\{(0.8, 1, 1.25); .2, .6, .3\}$, $\{(0, 0,0); .2, .6, .3\}$	$-$	$\{(0.29, 0.75, 2.43); .2, .6, .3\}$
5	$\{(0.26, 0.75, 2.77); .2, .6, .3\}$	$\{(0.8, 1, 1.25); .2, .6, .3\}$, $\{(0, 0,0); .2, .6, .3\}$	$-$	$\{(0.26, 0.75, 2.77); .2, .6, .3\}$

$$SM = \sum_{i}^{3} \frac{Z_i^{Max}}{\lvert \varphi_i^{Max} \rvert} = \frac{\{(3.88, 17.33, 63.28); .2, .6, .3\}x_1 + \{(-27.75, -3.56, 15.29); .2, .6, .3\}x_2}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$SN = \sum_{4}^{5} \frac{Z_i^{Min}}{\lvert \varphi_i^{Min} \rvert} = \frac{\{(-28.68, -8, -1.74); .2, .6, .3\}\tilde{x}_1 + \{(-8.23, 0, 4.70); .2, .6, .3\}\tilde{x}_2}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

<u>**Arithmetic Average Technique:**</u>

$m_1 = \min\{\lvert \varphi_i^{Max} \rvert\};$ $\qquad \forall i = 1,2,\dots,r$

$m_1 = \{(0.11, 0.37, 1.38); .2, .6, .3\}$

$m_2 = \min\{\lvert \varphi_i^{Min} \rvert\};$ $\qquad \forall i = r+1, r+2, \dots, s$

$m_2 = \{(0.29, 0.75, 2.43); .2, .6, .3\}$

$$AV_2 = \frac{m_1 + m_2}{2} = \frac{\{(0.11, 0.37, 1.38); .2, .6, .3\} + \{(0.29, 0.75, 2.43); .2, .6, .3\}}{2}$$

$$= \frac{\{(0.4, 1.12, 3.81); .2, .6, .3\}}{2} = \{(0.2, 0.56, 1.9); .2, .6, .3\}$$

$$SM - SN = \frac{\begin{bmatrix} \{(3.88, 17.33, 63.28); .2, .6, .3\}x_1 + \{(-27.75, -3.56, 15.29); .2, .6, .3\}x_2 \\ -\{(-28.68, -8, -1.74); .2, .6, .3\}\tilde{x}_1 - \{(-8.23, 0, 4.70); .2, .6, .3\}\tilde{x}_2 \end{bmatrix}}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$= \frac{\{(5.62, 25.33, 91.96); .2, .6, .3\}x_1 + \{(-32.45, -3.56, 23.52); .2, .6, .3\}x_2}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ Z = \frac{SM - SN}{AV_2}$$

$$= \frac{\{(5.62, 25.33, 91.96); .2, .6, .3\}x_1 + \{(-32.45, -3.56, 23.52); .2, .6, .3\}x_2}{\{(0.2, 0.56, 1.9); .2, .6, .3\} \times \{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ Z = \frac{\{(2.95, 45.23, 45.98); .2, .6, .3\}x_1 + \{(-162.25, -6.35, 117.6); .2, .6, .3\}x_2}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,1,1); .4, .3, .3\}\tilde{x}_1 + \{(1,1,2); .6, .2, .2\}\tilde{x}_2 \leq \{(1,2,3); .6, .3, .1\}$$

$$\{(8,9,10); .2, .6, .2\}\tilde{x}_1 + \{(1,1,2); .4, .4, .3\}\tilde{x}_2 \leq \{(8,9,10); .3, .4, .3\}$$

$$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0); 1,0,0\}$$

The above problem is the NMOLFPP in Arithmetic Average Technique. After solving it, we obtain the optimal solution as

$$Max\ Z = \{(0.34, 11.3, 31.92); .2, .6, .3\}$$

$$x_1 = \{(0.8, 1, 1.25); .2, .6, .3\}$$

$$x_2 = \{(0, 0, 0); .2, .6, .3\}$$

New Arithmetic Average Technique:

$$m_1 = \min\{|\varphi_i^{Max}|\}; \qquad \forall i = 1,2, \dots, r$$

$$m_1 = \{(0.11, 0.37, 1.38); .2, .6, .3\}$$

$$m_2 = \min\{|\varphi_i^{Min}|\}; \qquad \forall i = r+1, r+2, \dots, s$$

$$m_2 = \{(0.29, 0.75, 2.43); .2, .6, .3\}$$

$$AV_5 = \frac{m_1 + m_2}{5} = \frac{\{(0.11, 0.37, 1.38); .2, .6, .3\} + \{(0.29, 0.75, 2.43); .2, .6, .3\}}{5}$$

$$= \frac{\{(0.4, 1.12, 3.81); .2, .6, .3\}}{5} = \{(0.08, 0.22, 0.76); .2, .6, .3\}$$

$$SM - SN = \frac{\begin{bmatrix} \{(3.88, 17.33, 63.28); .2, .6, .3\}x_1 + \{(-27.75, -3.56, 15.29); .2, .6, .3\}x_2 \\ -\{(-28.68, -8, -1.74); .2, .6, .3\}\tilde{x}_1 - \{(-8.23, 0, 4.70); .2, .6, .3\}\tilde{x}_2 \end{bmatrix}}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$= \frac{\{(5.62, 25.33, 91.96); .2, .6, .3\}x_1 + \{(-32.45, -3.56, 23.52); .2, .6, .3\}x_2}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ Z = \frac{SM - SN}{AV_5}$$

$$= \frac{\{(5.62, 25.33, 91.96); .2, .6, .3\}x_1 + \{(-32.45, -3.56, 23.52); .2, .6, .3\}x_2}{\{(0.08, 0.22, 0.76); .2, .6, .3\} \times \{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ Z = \frac{\{(7.39, 115.13, 1149.5); .2, .6, .3\}x_1 + \{(-42.69, -16.18, 294); .2, .6, .3\}x_2}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,1,1); .4, .3, .3\}\tilde{x}_1 + \{(1,1,2); .6, .2, .2\}\tilde{x}_2 \leq \{(1,2,3); .6, .3, .1\}$$

$$\{(8,9,10); .2, .6, .2\}\tilde{x}_1 + \{(1,1,2); .4, .4, .3\}\tilde{x}_2 \leq \{(8,9,10); .3, .4, .3\}$$

$$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0); 1,0,0\}$$

The above problem is the NMOLFPP in New Arithmetic Average Technique. After solving it, we obtain the optimal solution as

$$Max\ Z = \{(0.87, 28.78, 798.26); .2, .6, .3\}$$

$$x_1 = \{(0.8, 1, 1.25); .2, .6, .3\}$$

$$x_2 = \{(0, 0, 0); .2, .6, .3\}$$

Harmonic Average Technique:

$$H_{av_1} = H_{av}\left\{\left|\varphi_i^{Max}\right|\right\}, \qquad \forall i = 1,2, \dots, r$$

$$= \frac{N}{\sum_{i=1}^{3}\left|\varphi_i^{Max}\right|}; \qquad Here\ N = 3$$

$$= \{(0.2, 0.67, 2.45); .2, .6, .3\}$$

$$H_{av_2} = H_{av}\left\{\left|\varphi_i^{Max}\right|\right\}, \qquad \forall i = r+1, r+2, \dots, s$$

$$= \frac{N}{\sum_{i=4}^{5}\left|\varphi_i^{Max}\right|}; \qquad Here\ N = 2$$

$$= \{(0.27, 0.75, 2.59); .2, .6, .3\}$$

$$S_1 = \frac{SM}{H_{av_1}} = \frac{\{(3.88, 17.33, 63.28); .2, .6, .3\}x_1 + \{(-27.75, -3.56, 15.29); .2, .6, .3\}x_2}{\{(0.2, 0.67, 2.45); .2, .6, .3\} \times \{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$= \frac{\{(1.58, 25.86, 316.4); .2, .6, .3\}x_1 + \{(-11.32, -5.31, 6.24); .2, .6, .3\}x_2}{\{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$S_2 = \frac{SN}{H_{av_2}} = \frac{\{(-28.68, -8, -1.74); .2, .6, .3\}\tilde{x}_1 + \{(-8.23,0,4.70); .2, .6, .3\}\tilde{x}_2}{\{(0.27, 0.75, 2.59); .2, .6, .3\} \times \{(1,2,3); .3, .5, .2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$= \frac{\{(-106.22,-10.66,-0.67);.2,.6,.3\}\tilde{x}_1 + \{(-30.48,0,17.40);.2,.6,.3\}\tilde{x}_2}{\{(1,2,3);.3,.5,.2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ Z = S_1 - S_2$$

$$= \frac{\begin{bmatrix} \{(1.58,25.86,316.4);.2,.6,.3\}x_1 + \{(-11.32,-5.31,6.24);.2,.6,.3\}x_2 \\ -\{(-106.22,-10.66,-0.67);.2,.6,.3\}\tilde{x}_1 - \{(-30.48,0,17.40);.2,.6,.3\}\tilde{x}_2 \end{bmatrix}}{\{(1,2,3);.3,.5,.2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$= \frac{\{(2.25,36.66,422.62);.2,.6,.3\}\tilde{x}_1 + \{(-28.72,-5.31,36.72);.2,.6,.3\}\tilde{x}_2}{\{(1,2,3);.3,.5,.2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,1,1);.4,.3,.3\}\tilde{x}_1 + \{(1,1,2);.6,.2,.2\}\tilde{x}_2 \le \{(1,2,3);.6,.3,.1\}$$

$$\{(8,9,10);.2,.6,.2\}\tilde{x}_1 + \{(1,1,2);.4,.4,.3\}\tilde{x}_2 \le \{(8,9,10);.3,.4,.3\}$$

$$\tilde{x}_1, \tilde{x}_2 \ge \{(0,0,0);1,0,0\}$$

The above problem is the NMOLFPP in Harmonic Average Technique. After solving it, we obtain the optimal solution as

$$Max\ Z = \{(0.26,9.16,293.48);.2,.6,.3\}$$

$$x_1 = \{(0.8,1,1.25);.2,.6,.3\}$$

$$x_2 = \{(0,0,0);.2,.6,.3\}$$

<u>Advanced Harmonic Average Technique:</u>

$$m_1 = \min\{|\varphi_i^{Max}|\}; \qquad \forall i = 1,2,\dots,r$$

$$m_1 = \{(0.11,0.37,1.38);.2,.6,.3\}$$

$$m_2 = \max\{|\varphi_i^{Min}|\}; \qquad \forall i = r+1, r+2, \dots, s$$

$$m_2 = \{(0.26,0.75,2.77);.2,.6,.3\}$$

$$AH_{av} = \frac{2|m_1||m_2|}{|m_1| + |m_2|} = \frac{\{(0.04,0.54,7.64);.2,.6,.3\}}{\{(0.37,1.12,4.15);.2,.6,.3\}} = \{(0.009,0.48,20.64);.2,.6,.3\}$$

$$Max\ Z = \frac{SM - SN}{AH_{av}}$$

$$= \frac{\{(5.62,25.33,91.96);.2,.6,.3\}x_1 + \{(-32.45,-3.56,23.52);.2,.6,.3\}x_2}{\{(0.009,0.48,20.64);.2,.6,.3\} \times \{(1,2,3);.3,.5,.2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ Z = \frac{\{(0.27,52.77,10217.77);.2,.6,.3\}x_1 + \{(-3605.55,-7.41,2613.33);.2,.6,.3\}x_2}{\{(1,2,3);.3,.5,.2\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,1,1);.4,.3,.3\}\tilde{x}_1 + \{(1,1,2);.6,.2,.2\}\tilde{x}_2 \le \{(1,2,3);.6,.3,.1\}$$

$$\{(8,9,10);.2,.6,.2\}\tilde{x}_1 + \{(1,1,2);.4,.4,.3\}\tilde{x}_2 \le \{(8,9,10);.3,.4,.3\}$$

$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0); 1,0,0\}$

The above problem is the NMOLFPP in Advanced Harmonic Average Technique. After solving it, we obtain the optimal solution as

$Max\ Z = \{(0.03, 13.19, 7095.67); .2, .6, .3\}$

$x_1 = \{(0.8, 1, 1.25); .2, .6, .3\}$

$x_2 = \{(0, 0, 0); .2, .6, .3\}$

Table 2: Comparison between results of the Numerical results

S. No.	Techniques	Numerical Example
1	Arithmetic Average Technique	$\{(0.34, 11.3, 31.92); .2, .6, .3\}$
2	New Arithmetic Average Technique	$\{(0.87, 28.78, 798.26); .2, .6, .3\}$
3	Harmonic Average (H_{av}) Technique	$\{(0.26, 9.16, 293.48); .2, .6, .3\}$
4	Advanced Harmonic Average (AH_{av}) Technique	$\{(0.03, 13.19, 7095.67); .2, .6, .3\}$

5. Conclusion

In this research paper, we introduced and defined Arithmetic Average Technique, New Arithmetic Average Technique, and Harmonic Average Technique. We then compared the Advanced Harmonic Average Technique with other standard techniques such as Arithmetic Average, New Arithmetic Average, and Harmonic Average Techniques. The comparison was based on the value of the objective functions. Upon solving a numerical example, we discovered that the optimal solution provided by our technique (Advanced Harmonic Average) was superior.

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